

## MathExcel Supplemental Problems # 4: Derivative Practice

1. State two versions (one with  $x \rightarrow a$  and one with  $h \rightarrow 0$ ) of the limit definition of the derivative. With your group, sketch a graph for each. Discuss what is happening in terms of secant lines and tangent lines.
2. Use either (or both!) of the definitions to compute the derivatives of the following functions.

(a)  $f(x) = \frac{1}{x+8}$

(b)  $f(x) = \sqrt{x+2}$

(c)  $f(x) = ax^2 + bx + c$

3. Use the  $x \rightarrow a$  version of the definition to compute the derivative of  $f(x) = x^n$ , where  $n$  is any real number. The identity

$$x^n - a^n = (x - a)(x^{n-1} + ax^{n-2} + a^2x^{n-3} + \dots + a^{n-2}x + a^{n-1})$$

may be helpful.

4. Use the  $h \rightarrow 0$  version of the definition to compute the derivative of  $f(x) = \sin(x)$ . The identities

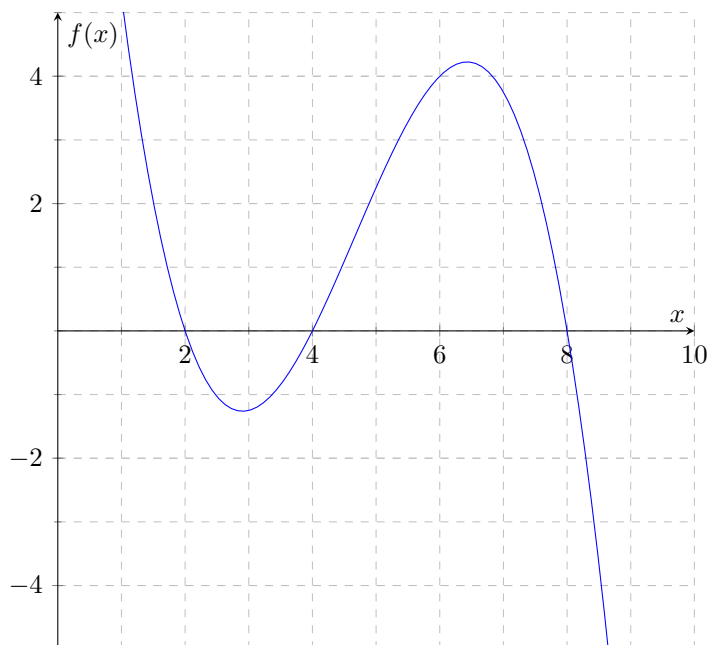
$$\sin(A+B) = \sin(A)\cos(B) + \sin(B)\cos(A),$$

$$\lim_{h \rightarrow 0} \frac{\sin(h)}{h} = 1, \text{ and}$$

$$\lim_{h \rightarrow 0} \frac{\cos(h) - 1}{h} = 0$$

will be useful.

5. Consider the function  $f(x)$  in the graph below.



- (a) Sketch the tangent line at  $x = 4$  and use this to estimate  $f'(4)$ .
- (b) Sketch the tangent line at  $x = 6$  and use this to estimate  $f'(6)$ .

6. Compute the following derivatives using shortcut rules for polynomials. You may need to rewrite some of the expressions first.

- (a)  $\frac{d}{dx} (x^4 - x^3 + \pi^2 - 1000)$
- (b)  $\frac{d}{dx} \left( \frac{2}{x} + 3x^3 + 4\sqrt{x} - 7x^{-1} + x \right)$
- (c)  $\frac{d}{dk} (7p^2 + 4k^2)$
- (d)  $\frac{d}{dt} \left( \frac{7t^5 + 4t^{-3}}{5t^2} \right)$

7. Let  $f(x) = x^2 + 5x + 7$

- (a) Find  $f'(x)$ .
- (b) Compute  $f(3)$  and  $f'(3)$ . Discuss what these values tell you about the graph of  $f$ .
- (c) Write an equation of the tangent line to  $f(x)$  at  $x = 3$ . Express your answer in point-slope form.

8. Find all values of  $x$  where  $f(x) = x^3$  and  $g(x) = 3x^2 + 30x + 2$  have parallel tangent lines.

9. Consider the function defined by

$$f(x) = \begin{cases} (x+1)^2 & x \leq 0 \\ Ax + B & x > 0 \end{cases}$$

where  $A$  and  $B$  are constants. Find values for  $A$  and  $B$  which make  $f(x)$  differentiable everywhere.

10. Consider the function defined by

$$f(x) = \begin{cases} ax + b & x < -2 \\ c & x = -2 \\ x^2 + 5 & x > -2 \end{cases}.$$

Find  $a$ ,  $b$ , and  $c$  such that  $f(x)$  is differentiable at  $x = -2$ .

11. Let  $f(x) = x^2 + ax + b$ .

- (a) Determine values of  $a$  and  $b$  such that  $f(1) = 8$  and  $f'(3) = 10$ .
- (b) Find an equation for the tangent line to  $f(x)$  at  $x = 2$ .

12. Let  $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x$ . Find the values of  $x$  such that  $f'(x) = 0$ .

13. Suppose that  $f(5) = 1$ ,  $g(5) = 3$ ,  $h(5) = 2$ ,  $f'(5) = 2$ ,  $g'(5) = 7$ , and  $h'(5) = -3$ . Let  $p$  be a constant. Use the derivative rules to determine  $k'(5)$  for each of the following functions.

- (a)  $k(x) = f(x) + 7h(x) - 3g(x)$
- (b)  $k(x) = f(x) - \frac{1}{3}g(x) + 4h(x)$
- (c)  $k(x) = pf(x) - \pi h(x) + 7g(x)$

14. Suppose that  $f(x) = \sqrt{x} + 3$ . Find the domain and range of  $f(x)$ . Find the derivative  $f'(x)$  and its domain and range.

15. Let  $f(x) = |x|$  and  $g(x) = x|x|$ .

- (a) Explain why  $f(x)$  is not differentiable at  $x = 0$ .
- (b) Sketch the graph of  $g(x)$ .
- (c) Determine  $g'(x)$ . In particular, what is  $g'(0)$ ?

16. The power that a battery supplies to a device depends on the internal resistance of the battery. For a battery of voltage  $V$  and an internal resistance  $r$ , the total power delivered to a device with resistance  $R$  is given by the formula:  $P = \frac{V^2 R}{(R + r)^2}$ . Assuming that  $V$  and  $R$  are constants, determine  $\frac{dP}{dr}$ .